June 2013 MA - C1

1) $\dfrac{7+\sqrt{5}}{\sqrt{5}-1}$

	$\sqrt{5}$	1
$\sqrt{5}$	5	$\sqrt{5}$
7	$7\sqrt{5}$	7

$$\frac{7+\sqrt{5}}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{5+7+7\sqrt{5}+\sqrt{5}}{\sqrt{5}^2-1^2}$$

Use difference of 2 squares to find denominator

$$= \frac{12+8\sqrt{5}}{4}$$

$$= 3+2\sqrt{5}$$

2) $\displaystyle\int \left(16x^4 - 4x - 3x^{\frac{1}{2}}\right) dx$

$$= 16\frac{x^5}{5} - 4\frac{x^2}{2} - 3\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2x^5 - 2x^2 - 6\sqrt{x} + c$$

3)

a) $8^{\frac{5}{3}} = \left(\sqrt[3]{8}\right)^5$

$= 2^5$

$= 32$

b) $\dfrac{\left(2x^{\frac{1}{2}}\right)^3}{4x^2} = \dfrac{2^3 x^{\frac{3}{2}}}{2^2 x^2}$

$= 2^{3-2} x^{\frac{3}{2}-2}$

$= 2x^{-\frac{1}{2}}$

$= \dfrac{2}{\sqrt{x}}$

$= \dfrac{2\sqrt{x}}{x}$

4)

$a_1 = 4$

a) $a_2 = k(4+2)$

$= 6k$

b) $a_3 = k(6k+2)$

$= 6k^2 + 2k$

$\sum[a_1, a_2, a_3] = 2$

$4 + 6k + (6k^2 + 2k) = 2$

$6k^2 + 8k + 4 = 2$

$3k^2 + 4k + 2 = 1$

$3k^2 + 4k + 1 = \emptyset$

$\frac{-4 \pm \sqrt{16-12}}{6} = \frac{-4 \pm 2}{6}$

$k = -\frac{1}{3}, -1$

5)

a) $2(3x+4) > 1-x$

$6x + 8 > 1-x$

$7x > -7$

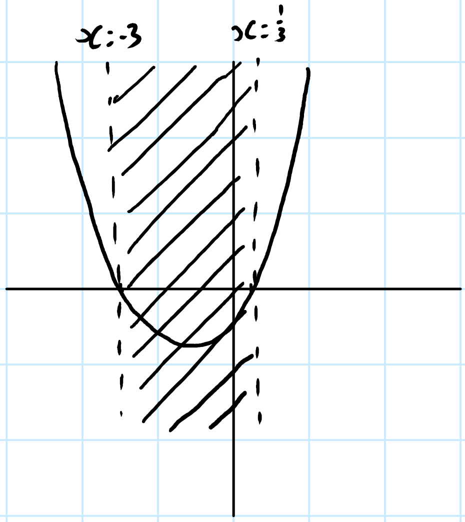
$x > -1$

b) $3x^2 + 8x - 3 < 0$

$$\frac{-8 \pm \sqrt{64 - -36}}{6} = \frac{-8 \pm 10}{6}$$

$x = \frac{1}{3}, -3$

$(3x-1)(x+3) < 0$



$-3 < x < \frac{1}{3}$

6)

Gradient of the line

a)

$$\frac{\Delta y}{\Delta x} = \frac{12-3}{11--1}$$

$$= \frac{9}{12}$$

$$= \frac{3}{4}$$

Using A as (x_1, y_1)

$$y-3 = \frac{3}{4}(x--1)$$

$$4y-12 = 3x+3$$

$$3x-4y+15 = 0$$

$a=3, b=-4, c=15$

6)

b) $3x - 4y + 15 = 0$, $3y + 4x - 30 = 0$

$9x - 12y + 45 = 0$, $12y + 16x - 120 = 0$

$12y = 9x + 45 \therefore (9x + 45) + 16x - 120 = 0$

$25x - 75 = 0$

$x - 3 = 0$

$x = 3$

$3(3) - 4y + 15 = 0$

$4y = 24$

$y = 6$

$(3, 6)$

7)

n = Week No

a)

$\therefore$ Phones per Week = $200 + (n-1)20$

$600 = 200 + (N-1)20$

$400 = 20N - 20$

$20 = N - 1$

$N = 21$

b) $S_{21} = \frac{1}{2} 21 (200 + 600)$

$= 400 \times 21$

$= 8400$

$$\begin{array}{r} 21 \\ \times 400 \\ \hline 8400 \end{array}$$

$600 \times (52-21) = 600 \times 31$

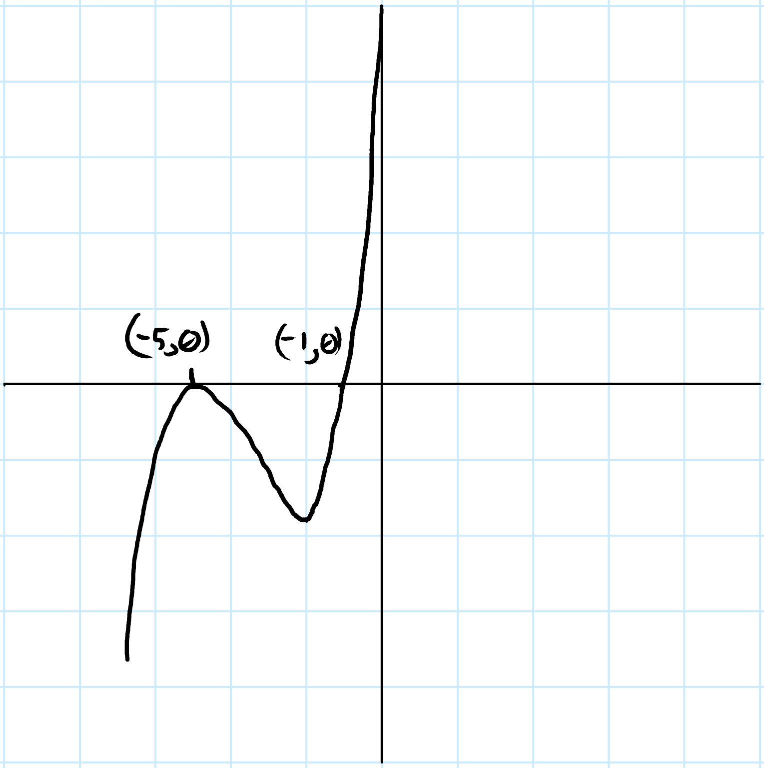
$= 18,600$

$$\begin{array}{r} 31 \\ \times 600 \\ \hline 8600 \\ +10000 \\ \hline 18,600 \end{array}$$

$8,400 + 18,600 = 27,000$ phones

8)

a)



b) $f(x+2) = (x+2+3)^2(x+2-1)$

$= (x+5)^2(x+1)$

c) $5^2 \times 1 = 25$

$y = 25$ when $x = 0$

9)

a) $f'(x) = \dfrac{(3 - x^2)^2}{x^2}$

$= \dfrac{3^2 + 2(3)(-x^2) + (-x^2)^2}{x^2}$

$= \dfrac{9}{x^2} - \dfrac{6x^2}{x^2} + \dfrac{x^4}{x^2}$

$= 9x^{-2} - 6 + x^2$

$A = -6, B = 1$

b) $f''(x) = \dfrac{d}{dx}\left[9x^{-2} - 6 + x^2\right]$

$= -18x^{-3} - 0 + 2x$

$= 2x - 18x^{-3}$

9)

c) $\int f'(x)\,dx = \int 4x^{-2} - 6 + x^2 \, dx$

$= \left[9\frac{x^{-1}}{-1} - 6\frac{x^1}{1} + \frac{x^3}{3} \right] + c$

$f(x) = -9x^{-1} - 6x + \frac{x^3}{3} + c$

given $f(-3) = 10$

$\frac{-9}{-3} - 6(-3) + \frac{-3^3}{3} + c = 10$

$3 + 18 - 9 + c = 10$

$c = -2$

$\therefore f(x) = -9x^{-1} - 6x + \frac{1}{3}x^3 - 2$

10)

$2x + y = 1$

$x^2 - 4ky + 5k = 0$

a)

$y = 1 - 2x$

$\therefore x^2 - 4k[1-2x] + 5k = 0$

$x^2 + 8kx + k = 0$

b) if $ax^2 + bx + c$ has equal roots

$b^2 - 4ac = 0$

$(8k)^2 - 4k = 0$

$16k^2 - k = 0$

$k \neq 0$ given in question

$\therefore k = \frac{1}{16}$

10(b) $x^2 + \frac{1}{2}x + \frac{1}{16} = 0$

c) $16x^2 + 8x + 1 = 0$

$\frac{-8 \pm \sqrt{64-64}}{32} = \frac{-8}{32}$

$x = -\frac{1}{4}$

$y = 1 - 2x$

$y = 1 - -\frac{2}{4}$

$= \frac{3}{2}$

$x = -\frac{1}{4}, \; y = \frac{3}{2}$

11)

a) $0 = \frac{3}{x} + 4$

$0 = 3 + 4x$

$\frac{-3}{4} = x$

b) $y = \frac{3}{(x+0)} + 4$

$\therefore \; x = 0 \; , \; y = 4$

c) $\frac{dy}{dx} = 3(-1)x^{-2} + 4(0)$

$= \frac{-3}{x^2}$

gradient at P $\frac{-3}{(-3)^2}$

$= -\frac{1}{3}$

gradient of Normal $+3$ $\qquad \because 3 \times \frac{-1}{3} = -1$

$y - y_1 = m(x - x_1)$

$y - 3 = 3(x - -3)$

$y - 3 = 3x + 9$

$y = 3x + 12$

11)

d)

$y = 3x + 12$

y intercept:

$y = 3(0) + 12$

$y = 12$

x intercept:

$0 = 3x + 12$

$-12 = 3x$

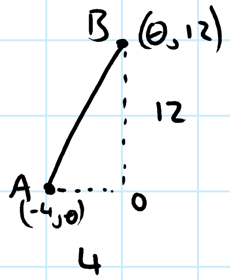
$x = -4$

OR

2 points are:

$B(0, 12)$

$A(-4, 0)$



$|AB| = \sqrt{4^2 + 12^2}$

$= \sqrt{16 + 144}$

$= \sqrt{160}$

$= 4\sqrt{10}$

$\overrightarrow{AB} = \begin{pmatrix} 0 \\ 12 \end{pmatrix} - \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 12 \end{pmatrix}$

$\left| \begin{matrix} 4 \\ 12 \end{matrix} \right| = \sqrt{12^2 + 4^2}$

$= \sqrt{(3 \times 4)^2 + (4)^2}$

$= \sqrt{4^2}\sqrt{3^2 + (1)^2}$

$= 4\sqrt{10}$